TABLA DE INTEGRALES

Formas básicas

1. $\displaystyle\int u\,dv = uv - \int v\,du$

2. $\displaystyle\int u^n\,du = \frac{u^{n+1}}{n+1} + C, \quad n \neq -1$

3. $\displaystyle\int \frac{du}{u} = \ln|u| + C$

4. $\displaystyle\int e^u\,du = e^u + C$

5. $\displaystyle\int a^u\,du = \frac{a^u}{\ln a} + C$

6. $\displaystyle\int \operatorname{sen} u\,du = -\cos u + C$

7. $\displaystyle\int \cos u\,du = \operatorname{sen} u + C$

8. $\displaystyle\int \sec^2 u\,du = \tan u + C$

9. $\displaystyle\int \csc^2 u\,du = -\cot u + C$

10. $\displaystyle\int \sec u\,\tan u\,du = \sec u + C$

11. $\displaystyle\int \csc u\,\cot u\,du = -\csc u + C$

12. $\displaystyle\int \tan u\,du = \ln|\sec u| + C$

13. $\displaystyle\int \cot u\,du = \ln|\operatorname{sen} u| + C$

14. $\displaystyle\int \sec u\,du = \ln|\sec u + \tan u| + C$

15. $\displaystyle\int \csc u\,du = \ln|\csc u - \cot u| + C$

16. $\displaystyle\int \frac{du}{\sqrt{a^2 - u^2}} = \operatorname{sen}^{-1}\frac{u}{a} + C, \quad a > 0$

17. $\displaystyle\int \frac{du}{a^2 + u^2} = \frac{1}{a}\tan^{-1}\frac{u}{a} + C$

18. $\displaystyle\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a}\sec^{-1}\frac{u}{a} + C$

19. $\displaystyle\int \frac{du}{a^2 - u^2} = \frac{1}{2a}\ln\left|\frac{u+a}{u-a}\right| + C$

20. $\displaystyle\int \frac{du}{u^2 - a^2} = \frac{1}{2a}\ln\left|\frac{u-a}{u+a}\right| + C$

Formas que involucran $\sqrt{a^2 + u^2}$, $a > 0$

21. $\displaystyle\int \sqrt{a^2 + u^2}\,du = \frac{u}{2}\sqrt{a^2 + u^2} + \frac{a^2}{2}\ln\!\left(u + \sqrt{a^2 + u^2}\right) + C$

22. $\displaystyle\int u^2\sqrt{a^2 + u^2}\,du = \frac{u}{8}(a^2 + 2u^2)\sqrt{a^2 + u^2} - \frac{a^4}{8}\ln\!\left(u + \sqrt{a^2 + u^2}\right) + C$

23. $\displaystyle\int \frac{\sqrt{a^2 + u^2}}{u}\,du = \sqrt{a^2 + u^2} - a\ln\left|\frac{a + \sqrt{a^2 + u^2}}{u}\right| + C$

24. $\displaystyle\int \frac{\sqrt{a^2 + u^2}}{u^2}\,du = -\frac{\sqrt{a^2 + u^2}}{u} + \ln\!\left(u + \sqrt{a^2 + u^2}\right) + C$

25. $\displaystyle\int \frac{du}{\sqrt{a^2 + u^2}} = \ln\!\left(u + \sqrt{a^2 + u^2}\right) + C$

26. $\displaystyle\int \frac{u^2\,du}{\sqrt{a^2 + u^2}} = \frac{u}{2}\sqrt{a^2 + u^2} - \frac{a^2}{2}\ln\!\left(u + \sqrt{a^2 + u^2}\right) + C$

27. $\displaystyle\int \frac{du}{u\sqrt{a^2 + u^2}} = -\frac{1}{a}\ln\left|\frac{\sqrt{a^2 + u^2} + a}{u}\right| + C$

28. $\displaystyle\int \frac{du}{u^2\sqrt{a^2 + u^2}} = -\frac{\sqrt{a^2 + u^2}}{a^2 u} + C$

29. $\displaystyle\int \frac{du}{(a^2 + u^2)^{3/2}} = \frac{u}{a^2\sqrt{a^2 + u^2}} + C$

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Formas que involucran $\sqrt{a^2 - u^2}$, $a > 0$

30. $\displaystyle \int \sqrt{a^2 - u^2}\, du = \frac{u}{2} \sqrt{a^2 - u^2} + \frac{a^2}{2} \operatorname{sen}^{-1} \frac{u}{a} + C$

31. $\displaystyle \int u^2 \sqrt{a^2 - u^2}\, du = \frac{u}{8} (2u^2 - a^2) \sqrt{a^2 - u^2} + \frac{a^4}{8} \operatorname{sen}^{-1} \frac{u}{a} + C$

32. $\displaystyle \int \frac{\sqrt{a^2 - u^2}}{u}\, du = \sqrt{a^2 - u^2} - a \ln \left| \frac{a + \sqrt{a^2 - u^2}}{u} \right| + C$

33. $\displaystyle \int \frac{\sqrt{a^2 - u^2}}{u^2}\, du = -\frac{1}{u} \sqrt{a^2 - u^2} - \operatorname{sen}^{-1} \frac{u}{a} + C$

34. $\displaystyle \int \frac{u^2\, du}{\sqrt{a^2 - u^2}} = -\frac{u}{2} \sqrt{a^2 - u^2} + \frac{a^2}{2} \operatorname{sen}^{-1} \frac{u}{a} + C$

35. $\displaystyle \int \frac{du}{u \sqrt{a^2 - u^2}} = -\frac{1}{a} \ln \left| \frac{a + \sqrt{a^2 - u^2}}{u} \right| + C$

36. $\displaystyle \int \frac{du}{u^2 \sqrt{a^2 - u^2}} = -\frac{1}{a^2 u} \sqrt{a^2 - u^2} + C$

37. $\displaystyle \int (a^2 - u^2)^{3/2}\, du = -\frac{u}{8} (2u^2 - 5a^2) \sqrt{a^2 - u^2} + \frac{3a^4}{8} \operatorname{sen}^{-1} \frac{u}{a} + C$

38. $\displaystyle \int \frac{du}{(a^2 - u^2)^{3/2}} = \frac{u}{a^2 \sqrt{a^2 - u^2}} + C$

Formas que involucran $\sqrt{u^2 - a^2}$, $a > 0$

39. $\displaystyle \int \sqrt{u^2 - a^2}\, du = \frac{u}{2} \sqrt{u^2 - a^2} - \frac{a^2}{2} \ln \left| u + \sqrt{u^2 - a^2} \right| + C$

40. $\displaystyle \int u^2 \sqrt{u^2 - a^2}\, du = \frac{u}{8} (2u^2 - a^2) \sqrt{u^2 - a^2} - \frac{a^4}{8} \ln \left| u + \sqrt{u^2 - a^2} \right| + C$

41. $\displaystyle \int \frac{\sqrt{u^2 - a^2}}{u}\, du = \sqrt{u^2 - a^2} - a \cos^{-1} \frac{a}{|u|} + C$

42. $\displaystyle \int \frac{\sqrt{u^2 - a^2}}{u^2}\, du = -\frac{\sqrt{u^2 - a^2}}{u} + \ln \left| u + \sqrt{u^2 - a^2} \right| + C$

43. $\displaystyle \int \frac{du}{\sqrt{u^2 - a^2}} = \ln \left| u + \sqrt{u^2 - a^2} \right| + C$

44. $\displaystyle \int \frac{u^2\, du}{\sqrt{u^2 - a^2}} = \frac{u}{2} \sqrt{u^2 - a^2} + \frac{a^2}{2} \ln \left| u + \sqrt{u^2 - a^2} \right| + C$

45. $\displaystyle \int \frac{du}{u^2 \sqrt{u^2 - a^2}} = \frac{\sqrt{u^2 - a^2}}{a^2 u} + C$

46. $\displaystyle \int \frac{du}{(u^2 - a^2)^{3/2}} = -\frac{u}{a^2 \sqrt{u^2 - a^2}} + C$

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Formas que involucran $a + bu$

47. $\displaystyle \int \frac{u\,du}{a + bu} = \frac{1}{b^2}\left(a + bu - a\ln|a + bu|\right) + C$

48. $\displaystyle \int \frac{u^2\,du}{a + bu} = \frac{1}{2b^3}\left[(a + bu)^2 - 4a(a + bu) + 2a^2\ln|a + bu|\right] + C$

49. $\displaystyle \int \frac{du}{u(a + bu)} = \frac{1}{a}\ln\left|\frac{u}{a + bu}\right| + C$

50. $\displaystyle \int \frac{du}{u^2(a + bu)} = -\frac{1}{au} + \frac{b}{a^2}\ln\left|\frac{a + bu}{u}\right| + C$

51. $\displaystyle \int \frac{u\,du}{(a + bu)^2} = \frac{a}{b^2(a + bu)} + \frac{1}{b^2}\ln|a + bu| + C$

52. $\displaystyle \int \frac{du}{u(a + bu)^2} = \frac{1}{a(a + bu)} - \frac{1}{a^2}\ln\left|\frac{a + bu}{u}\right| + C$

53. $\displaystyle \int \frac{u^2\,du}{(a + bu)^2} = \frac{1}{b^3}\left(a + bu - \frac{a^2}{a + bu} - 2a\ln|a + bu|\right) + C$

54. $\displaystyle \int u\sqrt{a + bu}\,du = \frac{2}{15b^2}(3bu - 2a)(a + bu)^{3/2} + C$

55. $\displaystyle \int \frac{u\,du}{\sqrt{a + bu}} = \frac{2}{3b^2}(bu - 2a)\sqrt{a + bu} + C$

56. $\displaystyle \int \frac{u^2\,du}{\sqrt{a + bu}} = \frac{2}{15b^3}(8a^2 + 3b^2u^2 - 4abu)\sqrt{a + bu} + C$

57. $\displaystyle \int \frac{du}{u\sqrt{a + bu}} = \frac{1}{\sqrt{a}}\ln\left|\frac{\sqrt{a + bu} - \sqrt{a}}{\sqrt{a + bu} + \sqrt{a}}\right| + C, \quad \text{si } a > 0$

$\displaystyle \qquad\qquad = \frac{2}{\sqrt{-a}}\tan^{-1}\sqrt{\frac{a + bu}{-a}} + C, \qquad \text{si } a < 0$

58. $\displaystyle \int \frac{\sqrt{a + bu}}{u}\,du = 2\sqrt{a + bu} + a\int \frac{du}{u\sqrt{a + bu}}$

59. $\displaystyle \int \frac{\sqrt{a + bu}}{u^2}\,du = -\frac{\sqrt{a + bu}}{u} + \frac{b}{2}\int \frac{du}{u\sqrt{a + bu}}$

60. $\displaystyle \int u^n\sqrt{a + bu}\,du = \frac{2}{b(2n + 3)}\left[u^n(a + bu)^{3/2} - na\int u^{n-1}\sqrt{a + bu}\,du\right]$

61. $\displaystyle \int \frac{u^n\,du}{\sqrt{a + bu}} = \frac{2u^n\sqrt{a + bu}}{b(2n + 1)} - \frac{2na}{b(2n + 1)}\int \frac{u^{n-1}\,du}{\sqrt{a + bu}}$

62. $\displaystyle \int \frac{du}{u^n\sqrt{a + bu}} = -\frac{\sqrt{a + bu}}{a(n - 1)u^{n-1}} - \frac{b(2n - 3)}{2a(n - 1)}\int \frac{du}{u^{n-1}\sqrt{a + bu}}$

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Formas trigonométricas

63. $\displaystyle\int \operatorname{sen}^2 u\, du = \tfrac{1}{2}u - \tfrac{1}{4}\operatorname{sen} 2u + C$

64. $\displaystyle\int \cos^2 u\, du = \tfrac{1}{2}u + \tfrac{1}{4}\operatorname{sen} 2u + C$

65. $\displaystyle\int \tan^2 u\, du = \tan u - u + C$

66. $\displaystyle\int \cot^2 u\, du = -\cot u - u + C$

67. $\displaystyle\int \operatorname{sen}^3 u\, du = -\tfrac{1}{3}(2 + \operatorname{sen}^2 u)\cos u + C$

68. $\displaystyle\int \cos^3 u\, du = \tfrac{1}{3}(2 + \cos^2 u)\operatorname{sen} u + C$

69. $\displaystyle\int \tan^3 u\, du = \tfrac{1}{2}\tan^2 u + \ln|\cos u| + C$

70. $\displaystyle\int \cot^3 u\, du = -\tfrac{1}{2}\cot^2 u - \ln|\operatorname{sen} u| + C$

71. $\displaystyle\int \sec^3 u\, du = \tfrac{1}{2}\sec u \tan u + \tfrac{1}{2}\ln|\sec u + \tan u| + C$

72. $\displaystyle\int \csc^3 u\, du = -\tfrac{1}{2}\csc u \cot u + \tfrac{1}{2}\ln|\csc u - \cot u| + C$

73. $\displaystyle\int \operatorname{sen}^n u\, du = -\frac{1}{n}\operatorname{sen}^{n-1} u \cos u + \frac{n-1}{n}\int \operatorname{sen}^{n-2} u\, du$

74. $\displaystyle\int \cos^n u\, du = \frac{1}{n}\cos^{n-1} u \operatorname{sen} u + \frac{n-1}{n}\int \cos^{n-2} u\, du$

75. $\displaystyle\int \tan^n u\, du = \frac{1}{n-1}\tan^{n-1} u - \int \tan^{n-2} u\, du$

76. $\displaystyle\int \cot^n u\, du = \frac{-1}{n-1}\cot^{n-1} u - \int \cot^{n-2} u\, du$

77. $\displaystyle\int \sec^n u\, du = \frac{1}{n-1}\tan u \sec^{n-2} u + \frac{n-2}{n-1}\int \sec^{n-2} u\, du$

78. $\displaystyle\int \csc^n u\, du = \frac{-1}{n-1}\cot u \csc^{n-2} u + \frac{n-2}{n-1}\int \csc^{n-2} u\, du$

79. $\displaystyle\int \operatorname{sen} au \operatorname{sen} bu\, du = \frac{\operatorname{sen}(a-b)u}{2(a-b)} - \frac{\operatorname{sen}(a+b)u}{2(a+b)} + C$

80. $\displaystyle\int \cos au \cos bu\, du = \frac{\operatorname{sen}(a-b)u}{2(a-b)} + \frac{\operatorname{sen}(a+b)u}{2(a+b)} + C$

81. $\displaystyle\int \operatorname{sen} au \cos bu\, du = -\frac{\cos(a-b)u}{2(a-b)} - \frac{\cos(a+b)u}{2(a+b)} + C$

82. $\displaystyle\int u \operatorname{sen} u\, du = \operatorname{sen} u - u \cos u + C$

83. $\displaystyle\int u \cos u\, du = \cos u + u \operatorname{sen} u + C$

84. $\displaystyle\int u^n \operatorname{sen} u\, du = -u^n \cos u + n\int u^{n-1}\cos u\, du$

85. $\displaystyle\int u^n \cos u\, du = u^n \operatorname{sen} u - n\int u^{n-1}\operatorname{sen} u\, du$

86. $\displaystyle\int \operatorname{sen}^n u \cos^m u\, du = -\frac{\operatorname{sen}^{n-1} u \cos^{m+1} u}{n+m} + \frac{n-1}{n+m}\int \operatorname{sen}^{n-2} u \cos^m u\, du$

$\displaystyle\qquad = \frac{\operatorname{sen}^{n+1} u \cos^{m-1} u}{n+m} + \frac{m-1}{n+m}\int \operatorname{sen}^n u \cos^{m-2} u\, du$

Formas trigonométricas inversas

87. $\displaystyle\int \operatorname{sen}^{-1} u\, du = u \operatorname{sen}^{-1} u + \sqrt{1-u^2} + C$

88. $\displaystyle\int \cos^{-1} u\, du = u \cos^{-1} u - \sqrt{1-u^2} + C$

89. $\displaystyle\int \tan^{-1} u\, du = u \tan^{-1} u - \tfrac{1}{2}\ln(1+u^2) + C$

90. $\displaystyle\int u \operatorname{sen}^{-1} u\, du = \frac{2u^2-1}{4}\operatorname{sen}^{-1} u + \frac{u\sqrt{1-u^2}}{4} + C$

91. $\displaystyle\int u \cos^{-1} u\, du = \frac{2u^2-1}{4}\cos^{-1} u - \frac{u\sqrt{1-u^2}}{4} + C$

92. $\displaystyle\int u \tan^{-1} u\, du = \frac{u^2+1}{2}\tan^{-1} u - \frac{u}{2} + C$

93. $\displaystyle\int u^n \operatorname{sen}^{-1} u\, du = \frac{1}{n+1}\left[u^{n+1}\operatorname{sen}^{-1} u - \int \frac{u^{n+1}\, du}{\sqrt{1-u^2}}\right], \quad n \neq -1$

94. $\displaystyle\int u^n \cos^{-1} u\, du = \frac{1}{n+1}\left[u^{n+1}\cos^{-1} u + \int \frac{u^{n+1}\, du}{\sqrt{1-u^2}}\right], \quad n \neq -1$

95. $\displaystyle\int u^n \tan^{-1} u\, du = \frac{1}{n+1}\left[u^{n+1}\tan^{-1} u - \int \frac{u^{n+1}\, du}{1+u^2}\right], \quad n \neq -1$

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Formas exponenciales y logarítmicas

96. $\displaystyle\int ue^{au}\,du = \frac{1}{a^2}(au-1)e^{au} + C$

97. $\displaystyle\int u^n e^{au}\,du = \frac{1}{a}u^n e^{au} - \frac{n}{a}\int u^{n-1}e^{au}\,du$

98. $\displaystyle\int e^{au}\operatorname{sen} bu\,du = \frac{e^{au}}{a^2+b^2}(a\operatorname{sen} bu - b\cos bu) + C$

99. $\displaystyle\int e^{au}\cos bu\,du = \frac{e^{au}}{a^2+b^2}(a\cos bu + b\operatorname{sen} bu) + C$

100. $\displaystyle\int \ln u\,du = u\ln u - u + C$

101. $\displaystyle\int u^n \ln u\,du = \frac{u^{n+1}}{(n+1)^2}\left[(n+1)\ln u - 1\right] + C$

102. $\displaystyle\int \frac{1}{u\ln u}\,du = \ln|\ln u| + C$

Formas hiperbólicas

103. $\displaystyle\int \operatorname{senh} u\,du = \cosh u + C$

104. $\displaystyle\int \cosh u\,du = \operatorname{senh} u + C$

105. $\displaystyle\int \tanh u\,du = \ln\cosh u + C$

106. $\displaystyle\int \coth u\,du = \ln|\operatorname{senh} u| + C$

107. $\displaystyle\int \operatorname{sech} u\,du = \tan^{-1}|\operatorname{senh} u| + C$

108. $\displaystyle\int \operatorname{csch} u\,du = \ln\left|\tanh\tfrac{1}{2}u\right| + C$

109. $\displaystyle\int \operatorname{sech}^2 u\,du = \tanh u + C$

110. $\displaystyle\int \operatorname{csch}^2 u\,du = -\coth u + C$

111. $\displaystyle\int \operatorname{sech} u\tanh u\,du = -\operatorname{sech} u + C$

112. $\displaystyle\int \operatorname{csch} u\coth u\,du = -\operatorname{csch} u + C$

Formas que involucran $\sqrt{2au-u^2}$, $a>0$

113. $\displaystyle\int \sqrt{2au-u^2}\,du = \frac{u-a}{2}\sqrt{2au-u^2} + \frac{a^2}{2}\cos^{-1}\!\left(\frac{a-u}{a}\right) + C$

114. $\displaystyle\int u\sqrt{2au-u^2}\,du = \frac{2u^2-au-3a^2}{6}\sqrt{2au-u^2} + \frac{a^3}{2}\cos^{-1}\!\left(\frac{a-u}{a}\right) + C$

115. $\displaystyle\int \frac{\sqrt{2au-u^2}}{u}\,du = \sqrt{2au-u^2} + a\cos^{-1}\!\left(\frac{a-u}{a}\right) + C$

116. $\displaystyle\int \frac{\sqrt{2au-u^2}}{u^2}\,du = -\frac{2\sqrt{2au-u^2}}{u} - \cos^{-1}\!\left(\frac{a-u}{a}\right) + C$

117. $\displaystyle\int \frac{du}{\sqrt{2au-u^2}} = \cos^{-1}\!\left(\frac{a-u}{a}\right) + C$

118. $\displaystyle\int \frac{u\,du}{\sqrt{2au-u^2}} = -\sqrt{2au-u^2} + a\cos^{-1}\!\left(\frac{a-u}{a}\right) + C$

119. $\displaystyle\int \frac{u^2\,du}{\sqrt{2au-u^2}} = -\frac{(u+3a)}{2}\sqrt{2au-u^2} + \frac{3a^2}{2}\cos^{-1}\!\left(\frac{a-u}{a}\right) + C$

120. $\displaystyle\int \frac{du}{u\sqrt{2au-u^2}} = -\frac{\sqrt{2au-u^2}}{au} + C$